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Three-dimensional heat transfer analysis of arrays of heated square blocks

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Abstract—Periodic fully developed heat transfer characteristics are computed for laminar flow through an array of heated square blocks deployed along one wall of a parallel-plate channel. This configuration simulates the forced-convection cooling of a printed circuit board encountered in electronic equipment. The thermal boundary condition to be considered is to assume that a specific amount of heat is generated uniformly at the bottom surface of each block. The calculations are performed for a range of Reynolds numbers, for a Prandtl number of 0.7, for several values of geometric and heated area parameters, and for two thermal conductivity ratios. The results are compared with the corresponding values for a parallel-plate duct and also with two limiting two-dimensional problems.

INTRODUCTION

In current electronic packaging designs, of low power rated electronic components on printed circuit boards, specific consideration is given to heat transfer analysis to achieve high heat dissipation rates and to limit peak temperature levels [1]. This concern has generated motivation for research studies into the forced-convection cooling of electronic equipment containing printed circuit boards. In most of the recent literature, an array of heated square blocks deployed along one wall of a parallel-plate channel is used to model the circuit board. Reference [2] reported a three-dimensional numerical analysis of this problem under the condition where the blocks were at a prescribed uniform wall temperature. Similar experimental investigations have been carried out by Sparrow *et al.* [3–5]. An extension of ref. [2] for a more realistic thermal boundary condition is given here. It is assumed that a specified amount of heat is uniformly generated at the bottom surface of each block. The numerical analysis is performed for the periodic fully developed region with the assumption of laminar flow. The results are presented in the form of surface temperature, heat flux, thermal resistance, and average Nusselt number for each block for a number of geometric and flow parameters. They will be compared with the corresponding values obtained for a parallel-plate duct and also with two limiting two-dimensional problems.

FORMULATION

Description of the problem

The problem to be considered in this study is schematically shown in Fig. 1. It involves the determination of three-dimensional heat transfer for laminar forced convection cooling of an array of square heat generating blocks. The blocks are deployed along one wall of a thermally insulated parallel walled duct. The configuration of the array of blocks is similar to the one considered in the authors' previous work [2]. However, a more realistic thermal boundary condition is considered here such that each block generates a specified amount of heat. Details of a heated block are schematically shown in Fig. 2. The heat is generated uniformly at the bottom surface of each block and is confined to the dotted square of dimension D.

The geometry of the problem is specified by the block dimension (L), the block thickness (B), the interblock gap (S), and the height of the flow passage between the block and the opposite wall of the channel (H). If L is chosen as a characteristic length, the dimensionless geometric parameters in the problem are B/L, S/L, and H/L.

The heat transfer and fluid flow will be analyzed for the periodic fully developed region where the solution domain is confined to a typical block shown by the dashed line in Fig. 1. The general concepts of this periodic fully-developed flow and heat transfer are discussed by Patankar *et al.* [6] and recently by Webb and Ramadhyani [7] for conjugate heat transfer problems. Therefore, only a brief description will be given here. Periodic fully-developed flow is characterized by a velocity field that repeats itself at corresponding axial stations in successive cycles. Furthermore, in such a regime, the pressure of cyclically corresponding locations decreases linearly in the downstream direction. Therefore, the pressure can be decomposed into two terms and is expressed by

$$p(x,y,z) = -\dot{p}z + p'(x,y,z) \qquad (1)$$

NOMENCLATURE

A_H	flow cross section above blocks
A_{surf}	block surface area, L^2+4BL
B	block thickness
c_p	specific heat of the fluid at constant pressure
D	dimension of heat generating area
H	height of open flow passage
h_m	average heat transfer coefficient (equation (14))
k_f	thermal conductivity of the fluid
k_s	thermal conductivity of the block
L	dimension of square block
$\dot{m}$	mass flow rate per spanwise width
Nu_m	average Nusselt number (equation (17))
N	dimensionless coordinate normal to the block surface
n	coordinate normal to the block
Pr	Prandtl number
P	dimensionless periodic pressure
p	pressure
p'	periodic component of pressure
$\dot{p}$	per-cycle pressure gradient
Q	heat transfer rate
q	heat flux
R	thermal resistance
Re	Reynolds number (equation (13))
S	intermodule gap
T'	dimensionless periodic component of temperature
$\bar{T}'$	average value of T'
t	temperature
t'	periodic component of temperature
$\bar{t}'$	average value of t'
U, V, W	dimensionless velocity components (equation (5))
u, v, w	velocity components
$\bar{W}$	dimensionless mean velocity w
$\bar{w}$	mean velocity (equation (12))
X, Y	dimensionless coordinates, $x/(L+S)$, $y/(L+S)$
x, y	coordinates
Z	dimensionless axial coordinate, $z/(L+S)$
z	axial coordinate.

Greek symbols

β	dimensionless per-cycle pressure gradient
μ	viscosity
ν	kinematic viscosity
ρ	density.

Subscripts

b	bulk value
bottom	bottom surface
high	highest value
surf	surface value.

where $\dot{p}$ is a constant pressure gradient, and the term $\dot{p}z$ represents the pressure drop that takes place in the flow direction. The quantity $p'(x, y, z)$ is the local pressure which behaves in a periodic manner from block to block.

Similarly, in a periodic thermally developed regime with the assumption of constant heat generation per block, the temperature of cyclically corresponding locations increases linearly in the downstream direction. Therefore, the temperature can be decomposed into two terms such that

$$t(x, y, z) = \sigma z + t'(x, y, z). \quad (2)$$

The term σz represents the non-periodic temperature rise where σ is a constant temperature gradient and is expressed as

$$\sigma = [t(x, y, z+L+S) - t(x, y, z)]/(L+S). \quad (3)$$

It may be shown that

$$\sigma = Q/\rho\bar{w}c_p A_H(L+S). \quad (4)$$

Furthermore, t' is the periodic temperature fluctuation which behaves in a periodic manner from block to block.

Conservation equations

The governing equations to be considered are the continuity, momentum, and energy equations. Constant thermophysical properties are assumed and natural convection is excluded. The following dimensionless variables are used:

$$X = x/(L+S), \quad Y = y/(L+S)$$
$$Z = z/(L+S), \quad U = u/(\nu/L+S)$$
$$V = v/(\nu/L+S), \quad W = w/(\nu/L+S)$$
$$P = p'/\rho(\nu/L+S)^2, \quad T' = t'/[Q/k_f(L+S)]$$
$$\beta = \dot{p}(L+S)/\rho(\nu/L+S)^2. \quad (5)$$

Then, upon the introduction of the dimensionless variables and parameters, the governing equations

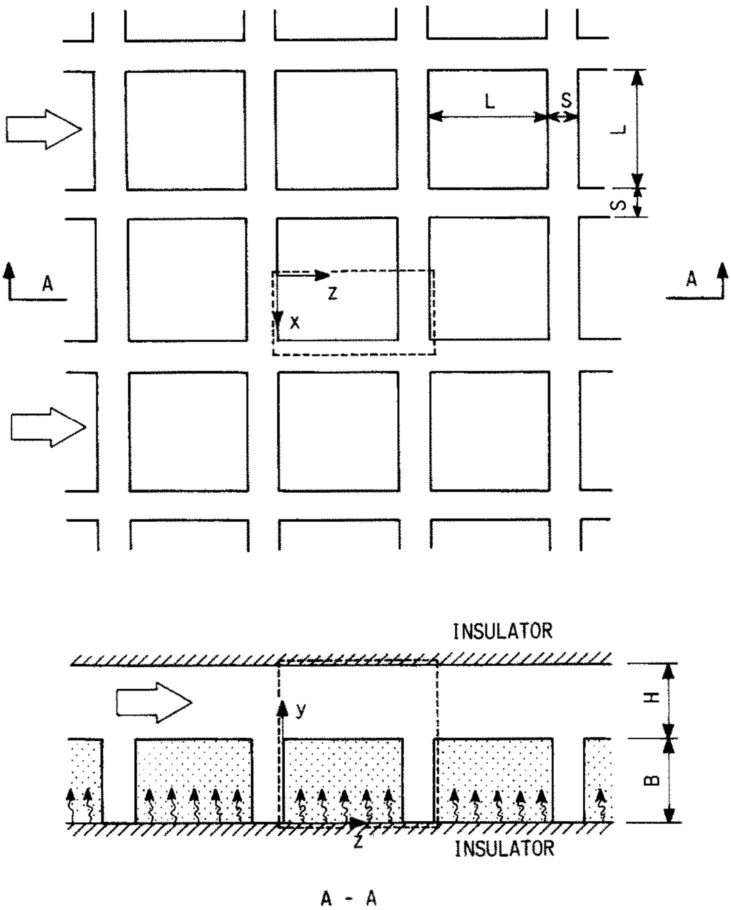

FIG. 1. Schematic diagram of an array of rectangular blocks deployed along one wall of a parallel-plate channel.

take the following form:

$$U(\partial U/\partial X)+V(\partial U/\partial Y)+W(\partial U/\partial Z) = \partial^2 U/\partial X^2+\partial^2 U/\partial Y^2+\partial^2 U/\partial Z^2-\partial P/\partial X \quad (6)$$

$$U(\partial V/\partial X)+V(\partial V/\partial Y)+W(\partial V/\partial Z) = \partial^2 V/\partial X^2+\partial^2 V/\partial Y^2+\partial^2 V/\partial Z^2-\partial P/\partial Y \quad (7)$$

$$U(\partial W/\partial X)+V(\partial W/\partial Y)+W(\partial W/\partial Z) = \partial^2 W/\partial X^2 +\partial^2 W/\partial Y^2+\partial^2 W/\partial Z^2-\partial P/\partial Z+\beta \quad (8)$$

$$U(\partial T'/\partial X)+V(\partial T'/\partial Y)+W(\partial T'/\partial Z) = (1/Pr)(\partial^2 T'/\partial X^2+\partial^2 T'/\partial Y^2+\partial^2 T'/\partial Z^2) - W/(Re\,Pr). \quad (9)$$

The last term on the right-hand side of equation (9) exists as a result of the decomposition of the temperature according to equation (2). The boundary conditions for the momentum equations (6)–(8) are $U = V = W = 0$ on the duct wall boundaries; $U = V = W = 0$ on the block wall boundaries, and $U = \partial V/\partial X = \partial W/\partial X = 0$ on the symmetry boundaries. At the inlet and outlet ends of the solution domain, periodic conditions are imposed. These are the equality of U, V, W, and P at the inlet and outlet boundaries.

The boundary conditions for energy equation (9) are as follows:

on the duct wall boundaries

$$\partial t/\partial y = 0$$

at the heat generating area

$$-k_s(\partial t/\partial y) = Q/D^2. \quad (10)$$

The corresponding dimensionless forms of equations (10) can be easily obtained as

$$\partial T'/\partial Y = 0$$

and

$$-(k_s/k_f)(\partial T'/\partial Y) = [(L+S)/D]^2. \quad (11)$$

At the inlet and outlet ends of the solution domain a periodic condition is imposed.

Having obtained the velocity field for a given parameter β, one can obtain the Reynolds number. This

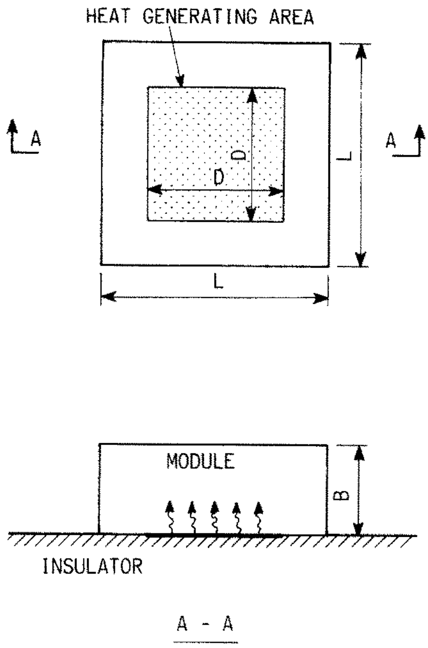

FIG. 2. Schematic diagram of a rectangular module.

is done by assuming the gap of height H between the blocks and the opposite wall of the channel as the main passage for fluid flow such that a characteristic velocity is evaluated by

$$\bar{w} = \dot{m}/A_H \tag{12}$$

where A_H is the flow cross section associated with the gap height H and $\dot{m}$ is the mass flow rate per spanwise width. Consistent with the foregoing definition, H will be selected as the characteristic dimension in the Reynolds number to obtain

$$Re = \rho\bar{w}H/\mu = \dot{m}/\mu(A_H/H)$$
$$= 2\int_0^{(H+B)/(L+S)}\int_0^{1/2} W\,\mathrm{d}X\,\mathrm{d}Y \tag{13}$$

where the quantity (A_H/H) is the spanwise width of the flow passage.

Numerical solution

Since the governing equations and boundary conditions for the fluid flow are the same as used in the previous paper [2], the computations for the velocity field will not be performed here. The previously calculated velocity field for Re in the range of 100–1000 will be used as an input to the energy equation. From the examination of governing equation (9) and boundary condition (11), it can be seen that there are three parameters which have to be specified prior to the initiation of the numerical calculations. These are the Prandtl number (Pr), the thermal conductivity ratio (k_s/k_f) for the block, and the geometric parameter of the heated area (D/L).

Epoxy resins and ceramic material are often used as sealant for electronic equipment [8]. The thermal conductivity of the epoxy resin ranges from 0.17 to 0.63 W m^{-1} K^{-1}, depending on the resin content. The thermal conductivity of a ceramic ranges from 16 to 24 W m^{-1} K^{-1}, depending on the aluminum content. In this paper, a value of 0.7 is selected for Pr, and selected values for (k_s/k_f) are 10 and 500. To investigate the effect of the size of the heat generating areas on heat transfer, two heated areas are examined. Selected values for D/L are 1 and 0.543. Note that in the case of $D/L = 1$, all the heat is generated at the bottom surface of the block.

Aside from Pr, k_s/k_f, and D/L, three geometric parameters B/L, S/L, and H/L need to be specified. The computations were carried out for $B/L = 3/8$, $S/L = 1/5$, 1/4, 1/3, 1/2, and $H/L = 1/4$, 3/8, 5/8 to investigate the effects of channel height and array density on heat transfer. These numerical experiments were performed systematically by varying S/L and H/L and the Reynolds number. The first set of runs was performed for $B/L = 3/8$, $S/L = 1/5$, and $H/L = 1/4$, for the four Reynolds numbers. These runs were repeated for $H/L = 3/8$ and 5/8. The next set of runs was similar to the above but for $S/L = 1/4$, 1/3, and 1/2. This required a total of 288 numerical experiments. The specific run for $B/L = 3/8$, $S/L = 1/4$, and $H/L = 5/8$ belongs to the dimensions used in the experiment by Sparrow *et al.* [3]. The ranges of geometric parameters are consistent with typical dimensions encountered in the printed circuit boards. Unfortunately, the experiment by Sparrow *et al.* [3] was performed for turbulent flow and therefore no comparisons with the experimental data will be made in this paper.

Computations were performed with $(16 \times 22 \times 30)$ grid points. These grid points were distributed in a non-uniform manner with higher concentration of grids closer to the walls. The grid size effect on the heat transfer and fluid flow were examined in the previous paper. Supplementary runs were performed with $(12 \times 16 \times 22)$ grid points to investigate the grid size effects for the case of $B/L = 3/8$, $S/L = 1/4$, and $H/L = 5/8$. Three values of the dimensionless pressure gradient β were selected such that the calculated Reynolds numbers ranged from 100 to 1000. The maximum change in the Nusselt number for the constant wall temperature thermal boundary between the coarse mesh and the fine mesh was within 0.3–5%.

The discretized procedure of the equations is based on the power-law scheme [9], and the discretized equations are solved by using a line-by-line method [10]. About 200–600 iterations were required to obtain a converged solution for the temperature field. The number of iterations depended on the geometric parameters and the Reynolds number.

It should be noted that in practice, the heat transfer may be influenced by buoyancy driven secondary flows at low Re. This is a topic of interest for further extension of the present work.

Nusselt number and heat flux

The parameters of interest in cooling of electronic equipment are the heat transfer coefficient on the block surface, the heat flux from the block surface, the thermal resistance in the block, and the highest temperature in the heat generating area. The average heat transfer coefficient on the block surface, h_m, is defined as

$$h_m = Q/A_{surf}(\overline{t'_{surf}} - \overline{t'_b}) \quad (14)$$

where $\overline{t'_{surf}}$ is the average surface temperature, and $\overline{t'_b}$ the average bulk temperature expressed by

$$\overline{t'_b} = \frac{1}{(L+S)} \int_0^{L+S} t'_b \, dz. \quad (15)$$

t'_b is the bulk temperature defined as

$$t'_b = \int_0^{H+B} \int_0^{(L+S)/2} wt' \, dx \, dy \bigg/ \int_0^{H+B} \int_0^{(L+S)/2} w \, dx \, dy. \quad (16)$$

The Nusselt number expression based on $(2H)$, is

$$Nu_m = h_m(2H)/k = \frac{Q(2H)}{A_{surf}k_f} \frac{1}{(\overline{t'_{surf}} - \overline{t'_b})} = \frac{(2H)(L+S)}{A_{surf}} \frac{1}{\overline{T'_{surf}} - \overline{T'_b}}. \quad (17)$$

The local heat flux on the block surface, q, is defined as

$$q = -k_f(\partial t/\partial n) \quad (18)$$

where n is the coordinate normal to the block surface. Utilizing equation (2), equation (18) can be rewritten as

$$q = -k_f[\sigma(\partial z/\partial n) + (\partial t'/\partial n)]. \quad (19)$$

Expressing it in dimensionless form, it can be expressed as

$$\frac{q}{Q/(L+S)^2} = -\frac{1}{Re\,Pr}\frac{\partial Z}{\partial N} - \frac{\partial T'}{\partial N} \quad (20)$$

where N is the dimensionless form of n. Depending on the direction of N, the term $(\partial Z/\partial N)$ takes one of three values: -1, 0, and 1.

RESULTS AND DISCUSSION

Surface temperature of block

The surface temperature of the rectangular block, $(\overline{T'_{surf}} - \overline{T'_b})$, corresponding to $k_s/k_f = 10$, $D/L = 0.543$, $Re = 982$, $H/L = 1/4$, $S/L = 1/4$, and $B/L = 3/8$ is shown in Fig. 3. In this figure, the surface temperature on the top, front, side, and rear of the block are shown in Figs. 3(a)–(d), respectively. The corners are marked with letters such as A, B,... in the block in Fig. 4(c). As seen from these figures, the temperature of the front, side, and rear surfaces decreases with elevation. The lowest and the highest temperatures appear at corners B and H, respectively. The deviations of the temperature at corners B and H from the average surface temperature $(\overline{T'_{surf}} - \overline{T'_b})$ are -40 and 35%, respectively. These deviations become large in the case of low channel height, and wide interblock gap.

Although the results are not presented here, the computations were also performed for the heated area parameter, $D/L = 1$. The heat generating area size mainly affects the bottom surface temperature. The effect of the heated area size on the surface temperature decreases with elevation. The computations were also performed for the high thermal conductivity parameters, $k_s/k_f = 500$, but the results are not presented here. In the case of high thermal conductivity, the deviation of the surface temperature from the average temperature becomes quite small. The deviation of the surface temperature from the average temperature ranges from -2 to 1% for all cases of $k_s/k_f = 500$.

The temperature at the bottom surface of the block for $D/L = 1$ and 0.543, corresponding to $k_s/k_f = 10$, and $Re = 982$, are shown in Figs. 4(a) and (b), respectively. The geometric parameters are $H/L = 1/4$, $S/L = 1/4$, and $B/L = 3/8$. The heated area is shaded in this figure. As seen in Fig. 4(b), in the case of $D/L = 0.543$, the temperature at the bottom surface of the heated area is high. On the contrary, in the case of $D/L = 1$, the temperature seems to be almost uniform. Although the results are not presented for the case of $k_s/k_f = 500$, the temperature at the bottom surface of the heat generating area is almost uniform for both cases of $D/L = 0.543$ and 1.

Heat flux from block surface

The generated heat at the bottom surface of the block is conducted through the block and is transferred to the fluid. This is a conjugate heat transfer problem. The heat flux from the block surface may not be uniform and is calculated as part of the solution. The heat flux from the block surface corresponding to $k_s/k_f = 10$, $D/L = 1$, and $Re = 982$; $k_s/k_f = 10$, $D/L = 0.543$, and $Re = 982$; $k_s/k_f = 500$, $D/L = 1$, and $Re = 982$ are shown in Figs. 5–7, respectively, for the geometric parameters $H/L = 1/4$, $S/L = 1/4$, and $B/L = 3/8$. In each figure, the heat flux from the top, front, side, and the rear surface of the block are shown in (a)–(d), respectively. The corners are marked with reference to Fig. 4(c). As seen from these figures, the heat flux at corners B and F are high. This tendency is accentuated in the case of high heat conductivity of the block (Fig. 7). As seen from Figs. 5(d) and 6(d), the heat flux from the rear surface takes a negative value. In these cases, the heat is transferred from the fluid to the block. In the case of $k_s/k_f = 500$ (Fig. 7), no inflow of the heat is observed.

Average Nusselt number

The average Nusselt number, defined by equation (17) is plotted as a function of the Reynolds number with the thermal conductivity ratio and the heated

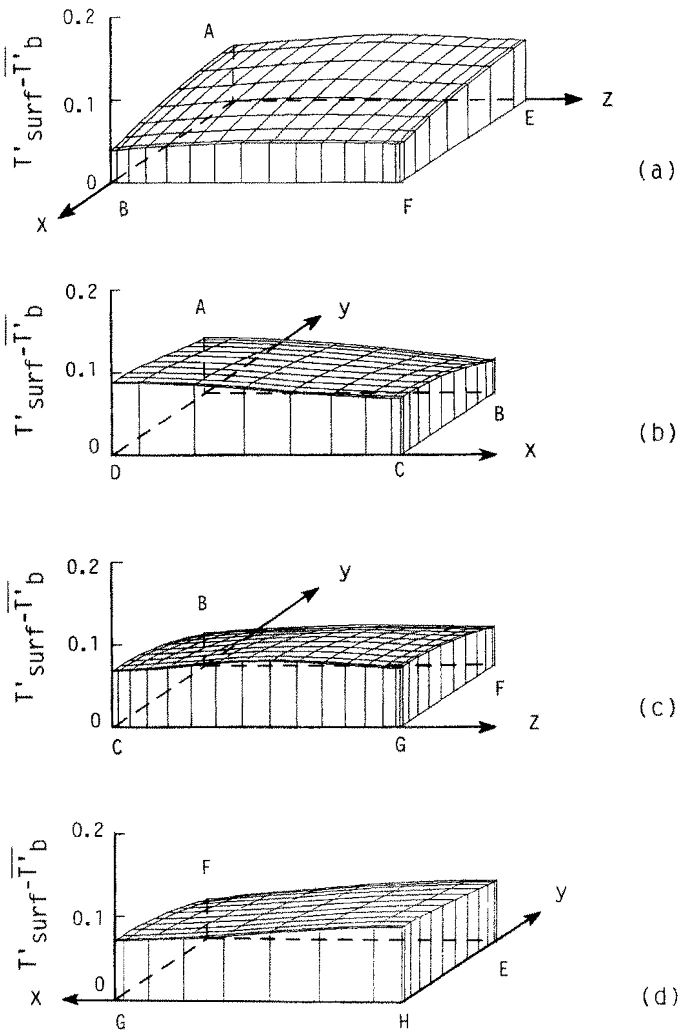

FIG. 3. Surface temperature corresponding to $k_s/k_f = 10$, $D/L = 0.543$, $Re = 982$, $B/L = 3/8$, $S/L = 1/4$, and $H/L = 1/4$ on (a) top, (b) front, (c) side, and (d) rear surfaces of a block.

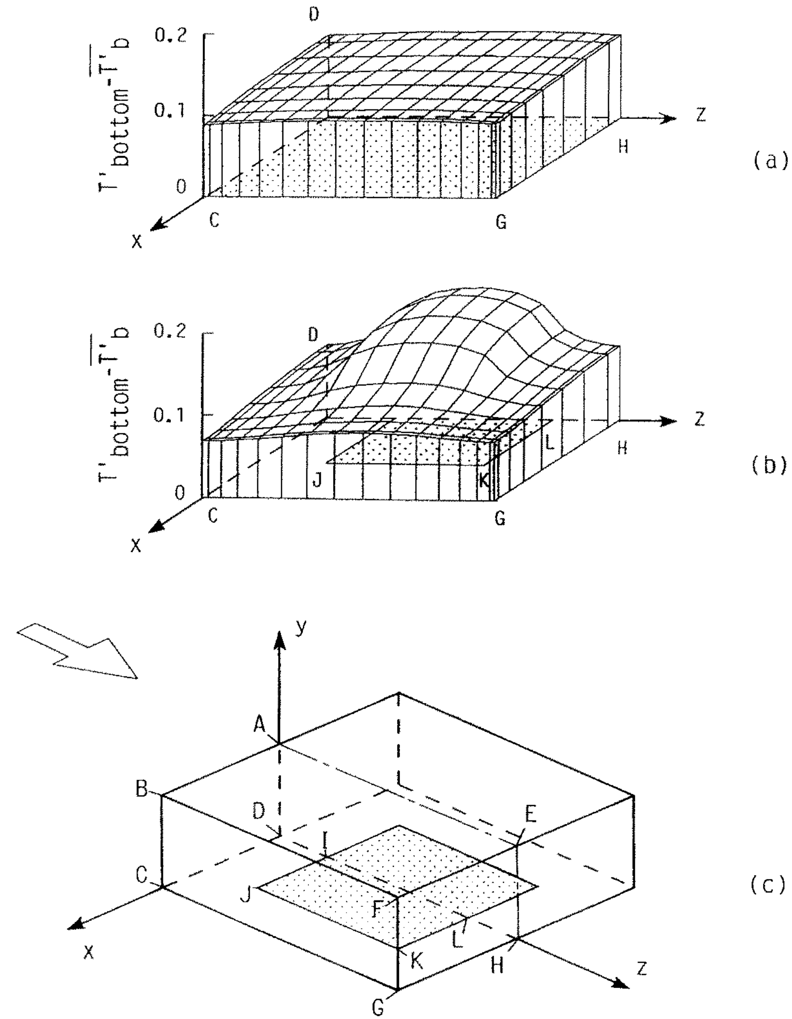

FIG. 4. Temperature at the bottom surface corresponding to $k_s/k_f = 10$, $Re = 982$, $B/L = 3/8$, $S/L = 1/4$, and $H/L = 1/4$ on (a) for $D/L = 1$, (b) for $D/L = 0.543$, and (c) schematic view of a block and the nomenclature used in Figs. 3–7.

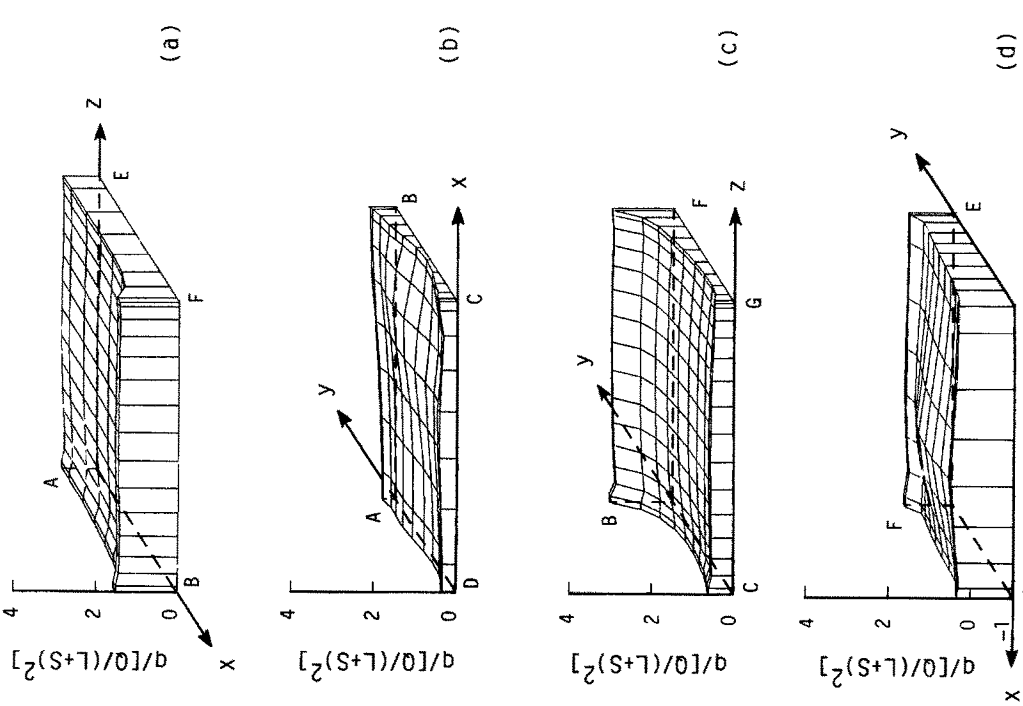

FIG. 5. Heat flux corresponding to $k_s/k_f = 10$, $D/L = 1$, $Re = 982$, $B/L = 3/8$, $S/L = 1/4$, $S/L = 1/4$, and $H/L = 1/4$ from a surface on the (a) top, (b) front, (c) side, and (d) rear of a block.

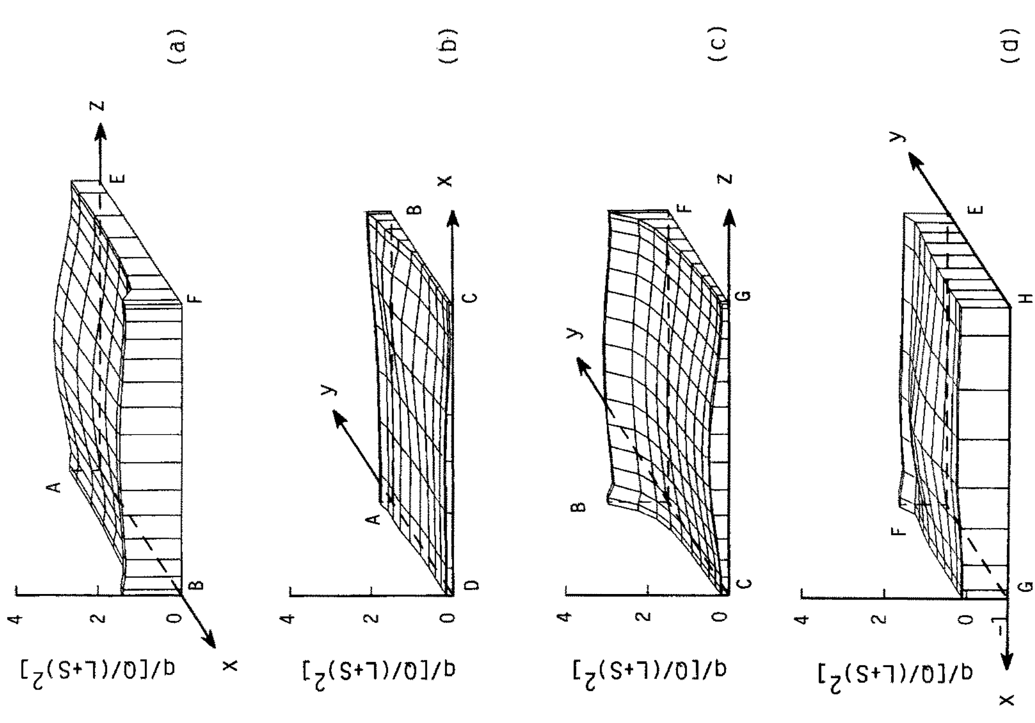

FIG. 6. Heat flux corresponding to $k_s/k_f = 10$, $D/L = 0.543$, $Re = 982$, $B/L = 3/8$, $S/L = 3/8$, $S/L = 1/4$, and $H/L = 1/4$ from a surface on the (a) top, (b) front, (c) side, and (d) rear of a block.

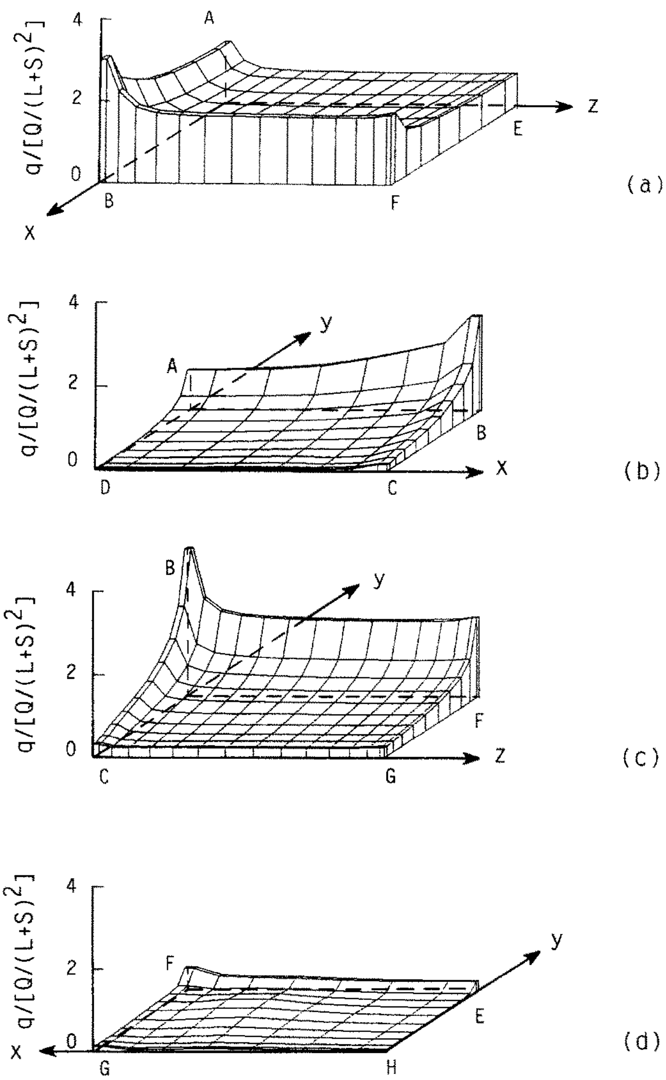

FIG. 7. Heat flux corresponding to $k_s/k_f = 500$, $D/L = 1$, $Re = 982$, $B/L = 3/8$, $S/L = 1/4$, and $H/L = 1/4$ from a surface on the (a) top, (b) front, (c) side, and (d) rear of a block.

area size as the curve parameters for a number of dimensionless interblock gaps ($S/L = 1/5$, $1/4$, $1/3$, and $1/2$) in Figs. 8–10, respectively. In each figure, the results for three values of the parameter H/L are plotted. The analytical value for a parallel-plate duct with one side insulated in the fully developed region is 5.385. This value is independent of Reynolds number and Prandtl number. Since the heat is mainly transferred from the top surface in the case of the square block, the Nusselt number is lower than the analytical value for the parallel-plate duct. The dashed and chain lines in the figures are the results of the two-dimensional calculations to be discussed later.

Thermal resistance

An expression for thermal resistance is often used in the design for cooling of electronic equipment. It is defined as

$$R_{HS} = (t'_{\text{high}} - \overline{t'_{\text{surf}}})/Q \tag{21}$$

where t'_{high} is the highest temperature in the heated area and $\overline{t'_{\text{surf}}}$ the average surface temperature. Equation (21) can be rewritten by using the dimensionless temperature as

$$R_{HS} = (T'_{\text{high}} - \overline{T'_{\text{surf}}})/k_f(L+S). \tag{22}$$

The representative temperature difference between the average surface temperature and the highest temperature in the heated area corresponding to the dimensionless interblock gap $S/L = 1/2$, the thermal conductivity ratio $k_s/k_f = 10$, and the heated area size $D/L = 1$ is plotted in Fig. 11 as a function of the Reynolds number with the dimensionless channel height H/L as the curve parameter. As seen from this

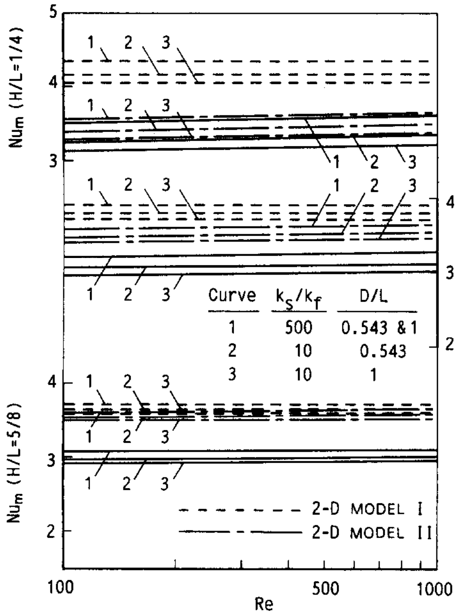

FIG. 8. Average Nusselt number as a function of Re for $S/L = 1/5$.

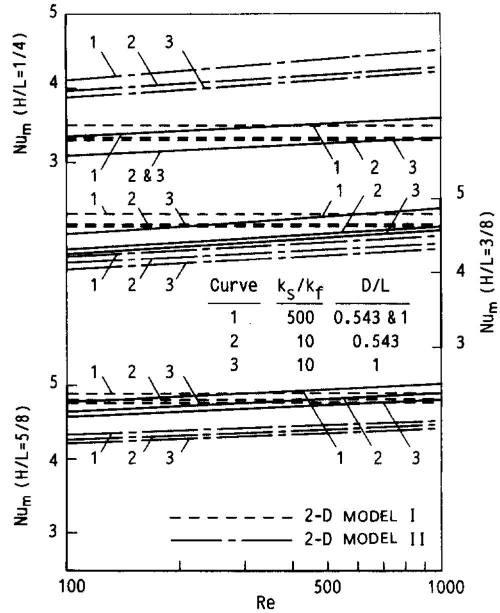

FIG. 10. Average Nusselt number as a function of Re for $S/L = 1/2$.

figure, the temperature difference is almost independent of Reynolds number and the dimensionless channel height, H/L. The temperature difference between the average surface temperature and the highest temperature for $Re = 1000$ and for the dimensionless channel height $H/L = 3/8$ is plotted in Fig. 12 as a function of the dimensionless interblock gap S/L with the thermal conductivity ratio k_s/k_f and the heat generating area size D/L as curve parameters.

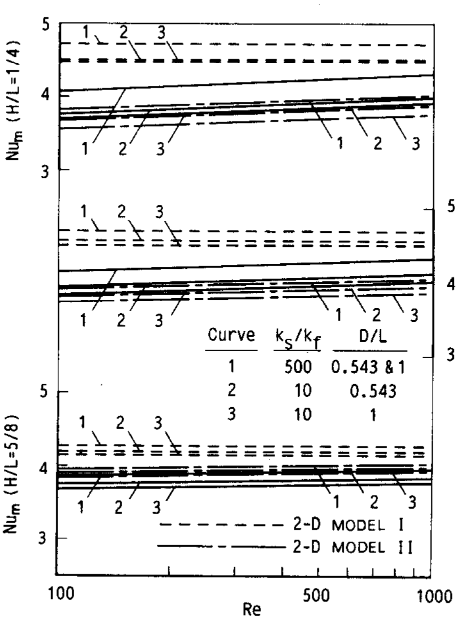

FIG. 9. Average Nusselt number as a function of Re for $S/L = 1/3$.

The block surface-to-fluid thermal resistance can be expressed as

$$R_{SA} = (\overline{t'_{\text{surf}}} - \overline{t'_b})/Q. \tag{23}$$

Equation (23) can be rewritten by using the Nusselt number defined by equation (17) as

$$R_{SA} = 1/[Nu_m(A_{\text{surf}}/2H)k_f]. \tag{24}$$

Then, the thermal resistance from the heat generating area to the fluid can be obtained from equations (22) and (24) as

$$R_{HA} = R_{HS} + R_{SA} = (T'_{\text{high}} - \overline{T'_{\text{surf}}})/k_f(L+S) + 1/[Nu_m(A_{\text{surf}}/2H)k_f]. \tag{25}$$

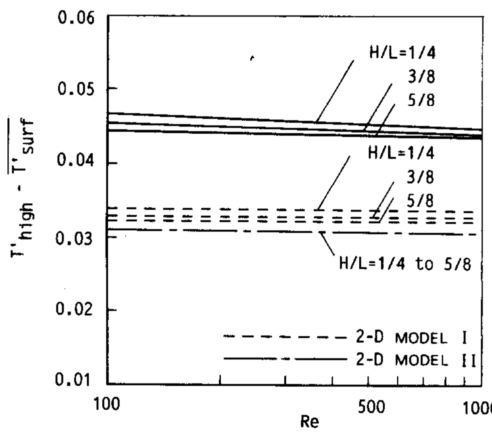

FIG. 11. Thermal resistance of block as a function of Re corresponding to $k_s/k_f = 10$, $D/L = 1$, and $S/L = 1/2$.

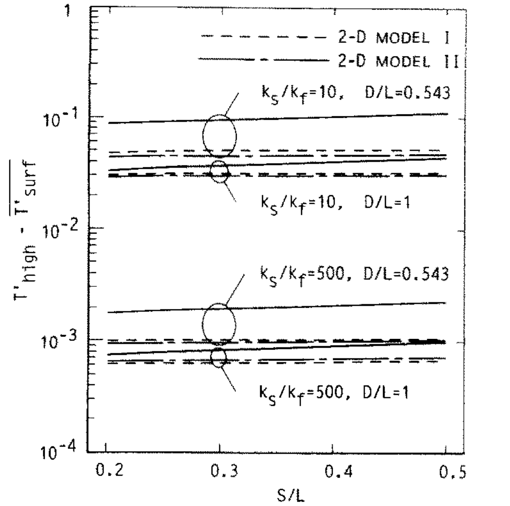

Fig. 12. Thermal resistance of block as a function of interblock gap corresponding to $H/L = 3/8$.

Comparisons with 2-D *blocks*

The full three-dimensional computation requires extensive computing time. Therefore, it would be helpful if the Nusselt number for the array of square blocks could be predicted from the result of a two-dimensional problem. To investigate this, supplementary two-dimensional computations were performed. Two models of the two-dimensional heat transfer block to be considered are schematically shown in Fig. 13. As seen there, the blocks have infinite length in one direction and are positioned successively along the wall. In the case of the two-dimensional block of model (I), the interblock gaps lie in the streamwise direction. In the case of model (II), the gaps lie in the spanwise direction to the flow. The geometry of the array is specified by the block length (L), the block thickness (B), the interblock gap (S), and the height of the flow passage between the block and the opposite wall of the channel (H). The solution domain is confined to a typical block which is speckled. The heat generating area which is positioned at the bottom of the block has infinite length in one direction and its width D is shaded in the figures.

The Nusselt number results are plotted by the dashed and chain lines in Figs. 8–10. The dashed line indicates the result for model (I), and is independent of Reynolds number. The chain line indicates the results for model (II), and is a function of Reynolds number. As seen from these figures, the results for model (I) predict higher Nusselt number values for the cases of $S/L = 1/5$, $1/4$, and $1/3$ and accurate values for the case of $S/L = 1/2$. On the contrary, good agreement between results for the square block and the two-dimensional model (II), is obtained for the case of $S/L = 1/3$.

The temperature differences between the average

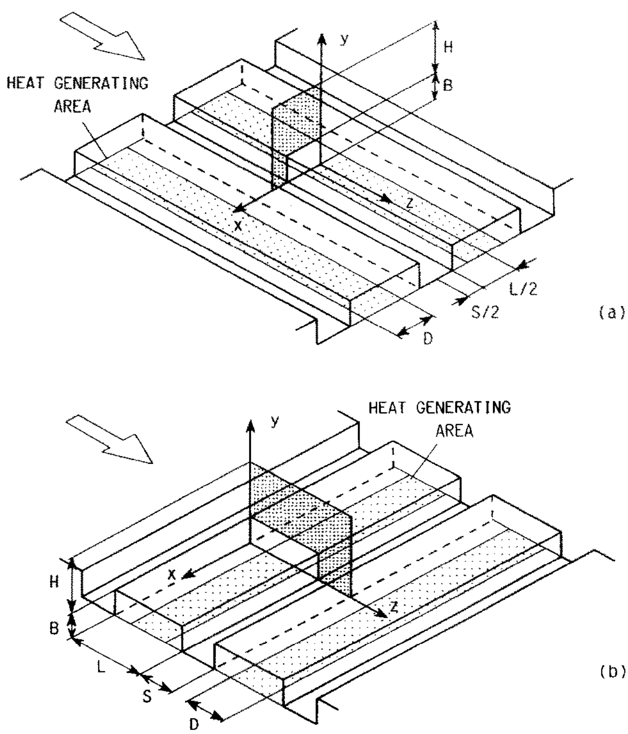

Fig. 13. (a) Schematic diagram of two-dimensional block of model (I). (b) Schematic diagram of two-dimensional block of model (II).

and highest temperatures are plotted by dashed and chain lines in Figs. 11 and 12. The dashed line indicates the result for model (I), and the chain line indicates the results for model (II). As seen from the figures, both values for models (I) and (II) predict low values.

CONCLUDING REMARKS

Periodic fully developed heat transfer and fluid flow characteristics are obtained for laminar flow through an array of heated square blocks deployed along one wall of a parallel plate channel. Heat is generated uniformly at the bottom surface of each block. The main conclusions of the results are given below.

(a) The surface temperature of each block is almost uniform for high values of thermal conductivity.

(b) The local heat flux on the top surface of the block is higher than that on the front, side, and rear surfaces of the block and the maximum values occur at the corners of the top surface.

(c) The average Nusselt number for an array is lower than the analytical value for the parallel-plate duct.

(d) The Nusselt number can be predicted by a two-dimensional model depending on the geometric parameters and whether the interblock gap lies in the streamwise or in the spanwise direction to the flow.

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ANALYSE DU TRANSFERT THERMIQUE TRIDIMENSIONNEL POUR UN ARRANGEMENT DE BLOCS CARRES ET CHAUDS

Résumé—On calcule les caractéristiques du transfert thermique périodique établi, pour un écoulement laminaire à travers un arrangement de blocs carrés chauds déployés le long d'une paroi d'un canal entre plaques parallèles, Cette configuration simule le refroidissement forcé d'un circuit imprimé dans un équipement électronique. La condition limite thermique est qu'une puissance spécifique est dégagée uniformément à la base de chaque bloc. Les calculs sont conduits pour un certain domaine de nombre de Reynolds, pour un nombre de Prandtl égal à 0,7 et pour différentes valeurs des paramètres géométriques et de surface chaude, ainsi que pour deux rapports de conductivité thermique. Les résultats sont comparés avec les valeurs correspondantes pour un canal à plaques parallèles et aussi avec les deux problèmes bidimensionnels limitants.

DER DREIDIMENSIONALE WÄRMEÜBERGANG AN ANORDNUNGEN VON BEHEIZTEN QUADRATISCHEN KÖRPERN

Zusammenfassung—Der periodisch vollständig entwickelte Wärmeübergang bei laminarer Strömung um ein Feld aus beheizten quadratischen Körpern, die an der Oberfläche eines durch zwei parallele Platten gebildeten Kanals angeordnet sind, wird berechnet. Diese Anordnung soll die Kühlung gedruckter Leiterbahnplatinen in elektronischen Geräten durch erzwungene Konvektion simulieren. Als thermische Randbedingung wird angenommen, daß die quadratischen Bauteile jeweils an der Unterseite gleichmäßig beheizt werden. Die Berechnungen wurden unter Variation der Reynolds-Zahl, der Bauteilgeometrie und der Heizflächenparameter bei einer Prandtl-Zahl von 0,7 für zwei Wärmeleitfähigkeitsverhältnisse durchgeführt. Die Ergebnisse werden mit den entsprechenden Werten für einen aus zwei parallelen Platten bestehenden Kanal und für zwei begrenzende zweidimensionale Probleme verglichen.